Composite Two-Higgs Models and Chiral Symmetry Restoration

A. A. Andrianov,[*]

Departament d'Estructura i Constituents de la Matèria,
Universitat de Barcelona,
Diagonal, 647, 08028 Barcelona, Spain,

V. A. Andrianov,[†]

Department of Theoretical Physics,
Institute of Physics, St.-Petersburg State University,
198904 St.-Petersburg, Russia,

R. Rodenberg,[‡]

Abteilung für Theoretische Elementarteilchen Physik,
III. Physikalisches Institut, Physikzentrum, RWTH-Aachen,
D-52056, Aachen, Germany

ABSTRACT: The effective quark models with quasilocal interaction are used for description of two composite Higgs doublets, in strong coupling (tricritical) regime below the compositeness scale Λ_C. The low energy effective action of Two-Higgs Doublet Standard Model (2HD SM) is obtained in the large N_c and large-log approximation. The two-point correlators of scalar and pseudoscalar Higgs fields are derived for investigation of how the chiral symmetry is broken. The comparison of their asymptotics at high energies allows to realize the chiral symmetry restoration characteristic for the QCD-like models and thereby to make hints on the existence of new physical phenomena in the TeV energy region.

KEYWORDS: Higgs Physics, Technicolor and Composite Models.

In Memory of Alexei A. Anselm

1. Introduction: definition of 2HQQM

This paper we dedicate to the memory of outstanding Russian physicist Alexei A. Anselm whose teaching and influence during many years was indispensable in our understanding of the Higgs phenomenon [1].

One of the minimal extensions of the Standard Model (SM) is the Two Higgs Doublet Model (2HDM) [2, 3] which has two complex doublets of Higgs bosons instead of only one in SM. The general 2HDM allows too strong flavour-changing neutral currents (FCNC) [4] as compared to the phenomenology of electroweak decays [5] . One possibility to suppress FCNC is to couple the fermions only to a fixed combination of the two Higgs doublets and its charge conjugated one which is know as a Model I [3]. There is another possibility to couple the first Higgs doublet to down-type quarks while the second one to up-type quarks, which is known as a Model II [4] . The physical content of the Higgs sector includes a pair of CP-even neutral scalar Higgs bosons, H^0 and h^0, a CP-odd neutral pseudoscalar Higgs boson A and a pair of charged Higgs bosons $H^{\pm}$. The mass spectrum of Models I and II has been extensively studied theoretically [2, 3, 6, 7] and bounded from the phenomenology of EW interactions at available high energies (for recent analyses, see [8, 9] and refs. therein).

If Higgs bosons are composite and their masses are created by a mechanism [10] of spontaneous chiral symmetry breaking [11, 12, 13, 14] below a scale of compositeness Λ one can require the chiral symmetry restoration (CSR) at high energies [15] which can lead to the CSR constraints on phenomenological parameters of a Higgs model. These constraints on Higgs boson masses (and other parameters) may serve to pinpoint the signatures of compositeness in the future experiments on Higgs particle observation [16].

Recently we have developed effective quark models including higher-dimensional operators made of fermion fields with derivatives, which can be used [17] for the parameterization of unknown heavy particle dynamics beyond SM in the spirit of Wilsonian effective action approach [18]. The inclusion of higher dimensional operators in the fermionic lagrangean of SM opens the ways to built the two (and more) Higgs doublet extension of SM with composite Higgs bosons.

In this paper we continue the exploration of particular Effective Quasilocal Quark Models (EQQM) [19] which inherit main properties of a underlying vector gauge theory of QCD type [20] (such as technicolor [21] or topcolor [22] models). The most important property turns out to be the Chiral Symmetry Breaking (CSB) at low energies and, on the other hand, the Chiral Symmetry Restoration (CSR) at high energies. The latter one is controlled by the Operator Product Expansion of quark current correlators which include a different number of parity-odd and parity-even currents. More specifically, here we deal with the two-point correlators of scalar and pseudoscalar quark densities [15, 23] which are saturated at low energies by Higgs-

particle resonances of a definite parity. The difference between these correlators is decreasing rapidly in accordance to OPE of a vector-like gauge theory [24, 36]. In the framework of either EQQM or a low-energy Higgs-field model it leads to the CSR constraints on some parameters of composite Higgs particles.

We show that CSR at high energies is indeed realized in the 2HQQM of type I in the Nambu-Jona-Lasinio phase near tricritical point and it is compatible with existence of relatively light scalar, h^0, and pseudoscalar, A, Higgs bosons. Thereby these models can have their origin from a QCD-like underlying theory with an electroweak compositeness scale of order $1 \div 10 TeV$.

Let us remind the effective quark lagrangian of a EQQM which incorporates all higher-dimensional operators necessary for the description of Two-Higgs Doublet Standard Model in the low-energy limit. The two-flavor, 3d generation quark models with quasilocal interaction are considered in which the t- and b-quarks are involved in the DCSB.

We restrict ourselves by examination of the Two- Higgs Quark Models of type I [19, 25] with Quasilocal interaction (2HQQM) with the following lagrangean:

$$\mathcal{L}_I = \bar{q}_L \not{D} q_L + \bar{t}_R \not{D} t_R + \bar{b}_R \not{D} b_R$$
$$+ \frac{1}{N_c \Lambda^2} \sum_{k,l=1}^{2} a_{kl} \left(g_{t,k} J_{t,k}^T + g_{b,k} \widetilde{J}_{b,k}^T \right) i\tau_2 \left(g_{b,l} J_{b,l} - g_{t,l} \widetilde{J}_{t,l} \right). \tag{1.1}$$

Here we have introduced the denotations for doublets of fermion currents:

$$J_{t,k} \equiv \bar{t}_R f_{t,k}(\hat{\tau}) q_L, \qquad J_{b,k} \equiv \bar{b}_R f_{b,k}(\hat{\tau}) q_L,, \qquad q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}, \tag{1.2}$$

and the tilde in $\widetilde{J}_{t,k}$ and $\widetilde{J}_{b,k}$ marks charge conjugated quark currents, rotated with τ_2 Pauli matrix

$$\widetilde{J}_{t,k} = i\tau_2 J_{t,k}^\star, \qquad \widetilde{J}_{b,k} = i\tau_2 J_{b,k}^\star \tag{1.3}$$

The subscripts t, b indicate right components of t and b quarks in the currents, the index k enumerates the formfactors:

$$f_{t,1}(\hat{\tau}) = 2 - 3\hat{\tau}; \quad f_{t,2}(\hat{\tau}) = -\sqrt{3}\hat{\tau}; \qquad \hat{\tau} \equiv -\frac{\partial^2}{\Lambda^2};$$
$$f_{b,1}(\hat{\tau}) = 2 - 3\hat{\tau}, \quad f_{b,2}(\hat{\tau}) = -\sqrt{3}\hat{\tau}; \tag{1.4}$$

which are orthonormal on the interval $0 \leq \tau = < \hat{\tau} > \leq 1$. In these notations coupling constants of the four-fermion interaction are represented by 2×2 matrix a_{kl} and contributed also from the Yukawa constants $g_{t,l}, g_{b,k}$.

2. Effective potential and mass spectrum of composite Higgs bosons

In order to describe the dynamics of composite Higgs bosons the lagrangean density (1.1) of the Model I must be rearranged by means of introduction of auxiliary

bosonic variables and by integrating out fermionic degrees of freedom [17]. Namely, we define two scalar $SU(2)_L$-isodoublets:

$$\Phi_1 = \begin{pmatrix} \phi_{11} \\ \phi_{12} \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} \phi_{21} \\ \phi_{22} \end{pmatrix} \tag{2.1}$$

and their charge conjugates:

$$\tilde{\Phi}_1 = \begin{pmatrix} \phi_{12}^\star \\ -\phi_{11}^\star \end{pmatrix}, \qquad \tilde{\Phi}_2 = \begin{pmatrix} \phi_{22}^\star \\ -\phi_{21}^\star \end{pmatrix}. \tag{2.2}$$

Then the lagrangean (1.1) can be rewritten in the following way:

$$\mathcal{L}_I = L_{kin} + N_c\Lambda^2 \sum_{k,l=1}^{2} \Phi_k^\dagger(a^{-1})_{kl}\Phi_l + i\sum_{k=1}^{2} \left[g_{t,k}\tilde{\Phi}_k^\dagger J_{t,k} + g_{b,k}\Phi_k^\dagger J_{b,k} \right] + h.c. \tag{2.3}$$

The integrating out of fermionic degrees of freedom will produce the effective action for Higgs bosons of which we shall keep only the kinetic term and the effective potential consisting of two- and four-particles operators. The omitted terms are supposedly small, being proportional to inverse powers of a large scale factor Λ and/or of a large $\ln(\Lambda/v)$; $v \simeq 246 GeV$. The Yukawa constants are chosen of the form

$$g_{t,k} = 1; \qquad g_{b,k} = g \tag{2.4}$$

for $k = 1, 2$. The first choice can be done because the fields Φ_1 and Φ_2 can always be rescaled by an arbitrary factor which is absorbed by redefinition of polycritical coupling constants a_{kl}. Other constants $g_{b,k}$ are taken equal for the simplicity. Their value g induces the quark mass ratio m_b/m_t.

A systematic approximation can be developed in the vicinity of (poly)critical point,

$$8\pi^2 a_{kl}^{-1} \sim \delta_{kl} + \frac{\Delta_{kl}}{\Lambda^2}, \qquad |\Delta_{kl}| \ll \Lambda^2, \tag{2.5}$$

which signifies the cancellation of quadratic divergences [26].

Then the effective potential of Higgs fields in 2HQQM of type I reads:

$$V_{eff} = \frac{N_c}{8\pi^2}\left(-\sum_{k,l=1}^{2}(\Phi_k^\dagger\Phi_l)\Delta_{kl} - 8g^4(\Phi_1^\dagger\Phi_1)^2 \ln g^2 \right.$$

$$+(1+g^4)\left[8(\Phi_1^\dagger\Phi_1)^2\left(\ln\frac{\Lambda^2}{4(\Phi_1^\dagger\Phi_1)} + \frac{1}{2} \right) + \right.$$

$$-\frac{159}{8}(\Phi_1^\dagger\Phi_1)^2 + \frac{9}{8}(\Phi_2^\dagger\Phi_2)^2 + \frac{3}{4}(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)+$$

$$+\frac{3}{4}(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{3}{8}(\Phi_1^\dagger\Phi_2)^2 + \frac{3}{8}(\Phi_2^\dagger\Phi_1)^2-$$

$$-\frac{5\sqrt{3}}{4}(\Phi_1^\dagger\Phi_1)\left((\Phi_1^\dagger\Phi_2) + (\Phi_2^\dagger\Phi_1) \right) +$$

$$\left.\left. +\frac{\sqrt{3}}{4}(\Phi_2^\dagger\Phi_2)\left((\Phi_1^\dagger\Phi_2) + (\Phi_2^\dagger\Phi_1) \right) \right] \right) + O\left(\frac{\ln\Lambda}{\Lambda^2} \right), \tag{2.6}$$

where the bilinear "mass" term is in general non-diagonal and represented by the real, symmetric 2×2 matrix Δ_{kl}. This is the two-Higgs potential in the large N_c approach and a more realistic potential should involve the true Renormalization Group flow of Two-Higgs Doublet SM [1] with initial conditions at high energies taken from (2.6).

In general there exist the regimes where at the minimum of the effective potential the ratio of v.e.v. of neutral Higgs fields is complex [26] and v.e.v. of charged components are also non-zero. Due to $U(2)$ symmetry of the effective potential (2.6) one can always choose the parameterization of Higgs fields in the vicinity of a minimum with only one of v.e.v. being complex:

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi_1 \end{pmatrix}, \qquad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \theta \\ \phi_2 + i\rho \end{pmatrix}. \tag{2.7}$$

The appearance of complex a v.e.v. leads to CP violation [2, 28, 29, 30] and a non-zero v.e.v. of charged fields breaks electric stability of the vacuum and supplies the photon with a mass. The latter phase is not realized in the Universe at zero temperatures as it follows from the severe bounds on both electric charge asymmetry and on the photon mass (see, e.g. [31]).

The condition of minimum of the potential (2.6) brings the mass-gap equations for them which solution may cause the DCSB if it is an absolute minimum. In the explicit form they are:

$$\frac{8\pi^2}{N_c} \frac{\delta V_{eff}(\phi_k, \rho, \theta)}{\delta \phi_1} = -\Delta_{11}\phi_1 - \Delta_{12}\phi_2 + 8\phi_1^3 \ln \frac{\Lambda^2}{2\phi_1^2} + 8\phi_1^3 g^4 \ln \frac{\Lambda^2}{2g^2\phi_1^2} \tag{2.8}$$

$$+ \frac{1+g^4}{8} \Big[-159\phi_1^3 - 15\sqrt{3}\phi_1^2\phi_2 + 9\phi_1\phi_2^2 + \sqrt{3}\phi_2^3$$

$$+ \sqrt{3}(\theta^2 + \rho^2)(\sqrt{3}\phi_1 + \phi_2) \Big] = 0,$$

$$\frac{8\pi^2}{N_c} \frac{\delta V_{eff}(\phi_k, \rho, \theta)}{\delta \phi_2} = -\Delta_{12}\phi_1 - \Delta_{22}\phi_2 + \frac{1+g^4}{8} \Big[-5\sqrt{3}\phi_1^3 + 9\phi_1^2\phi_2$$

$$+ 3\sqrt{3}\phi_1\phi_2^2 + 9\phi_2^3 + \sqrt{3}(\theta^2 + \rho^2)(\phi_1 + 3\sqrt{3}\phi_2) \Big] = 0$$

$$\frac{8\pi^2}{N_c} \frac{\delta V_{eff}(\phi_k, \rho, \theta)}{\delta \theta} = \theta \Big[-\Delta_{22} + \frac{1+g^4}{8} \Big(9(\theta^2 + \rho^2) + (\sqrt{3}\phi_1 + \phi_2)^2 + 8\phi_2^2 \Big) \Big] = 0;$$

$$\frac{8\pi^2}{N_c} \frac{\delta V_{eff}(\phi_k, \rho, \theta)}{\delta \rho} = \rho \Big[-\Delta_{22} + \frac{1+g^4}{8} \Big(9(\theta^2 + \rho^2) + (\sqrt{3}\phi_1 + \phi_2)^2 + 8\phi_2^2 \Big) \Big] = 0$$

We assume the electric charge stability of the vacuum, *i.e.* that only neutral components of both Higgs doublets may have nonzero v.e.v. [3, 6, 32]. Evidently, the relevant solutions with $\theta = \rho = 0$ are unique if

$$\Delta_{22} \leq (1 + g^4) \Big[\phi_2^2 + \frac{1}{8}(\sqrt{3}\phi_1 + \phi_2)^2 \Big]. \tag{2.9}$$

---

[1] see the updated one-loop RG equations in [27] and references therein.

4

This bound follows also from the absence of tachyons among composite bosons which will be derived later on. On the other hand, due to $SO(2)$ symmetry of effective potential (2.6) under rotations of θ and ρ, typically, the CP violating solutions for 2HQQM of type I break also the electroneutrality of the vacuum unless a special fine-tuning is performed. It gives us one more argument against the realization of CP-violating phase in such models.

One can choose ϕ_1, ϕ_2 as independent scales of 2HQQM. Then the mass-gap equations (2.9) allow to find the parameters Δ_{11}, Δ_{12} describing the deviation from critical coupling constants as functions of $\phi_1, \phi_2, \Delta_{22}$.

The true minimum is derived from the positivity of the second variation of the effective action around a solution of the mass-gap equation,

$$\Phi_k = \begin{pmatrix} \pi_k^+ \\ \frac{1}{\sqrt{2}}(\phi_k + \sigma_k + i\pi_k) \end{pmatrix}; \; k = 1, 2. \tag{2.10}$$

This variation reads:

$$\frac{16\pi^2}{N_c} S_{eff}^{(2)} \equiv \frac{1}{2} \left(\sigma, (\hat{A}^\sigma p^2 + \hat{B}^\sigma)\sigma \right)$$
$$+ \left(\pi^-, (\hat{A}^{\pi^+} p^2 + \hat{B}^{\pi^+})\pi^+ \right) + \frac{1}{2} \left(\pi, (\hat{A}^\pi p^2 + \hat{B}^\pi)\pi \right), \tag{2.11}$$

where two symmetric matrices - for the kinetic term $\hat{A}^i = (A_{kl}^i), \quad i = (\sigma, \pi^+, \pi)$ and for the constant, momentum independent part, $\hat{B}^i = (B_{kl}^i)$ - have been introduced in the CP conserving phase.

The mass spectrum of related bosonic states is determined by the solutions of the secular equations :

$$\det(\hat{A}^i p^2 + \hat{B}^i) = 0, \tag{2.12}$$

at $-m^2 = p^2 < 0$ in both scalar and pseudoscalar channels.

The kinetic matrix $\hat{A}$ as being multiplied by p^2 is derived in the soft-momentum expansion in powers of p^2:

$$\mathcal{L}_{\text{kin}} = \frac{N_c}{16\pi^2} \sum_{k,l=1}^2 \left(I_{kl}^{(1)} \partial_\mu \pi_k^- \partial_\mu \pi_l^+ + \frac{1}{2} I_{kl}^{(2)} (\partial_\mu \sigma_k \partial_\mu \sigma_l + \partial_\mu \pi_k \partial_\mu \pi_l) \right) \tag{2.13}$$

where $I_{kl}^{(1)}$ contributes to the kinetic term for charged components of Higgs doublets and $I_{kl}^{(2)}$ defines the latter one for the neutral components:

$$A_{kl}^\sigma \simeq A_{kl}^{\pi^+} \simeq A_{kl}^\pi \simeq I_{kl}^{(1)} \simeq I_{kl}^{(2)}$$
$$\simeq f_{t,k}(0)f_{t,l}(0) \left(\ln \frac{\Lambda^2}{m_t^2} - 1 \right) + g^2 f_{b,k}(0)f_{b,l}(0) \left(\ln \frac{\Lambda^2}{m_b^2} - 1 \right)$$
$$+ \int_0^1 (f_{t,k}(\tau)f_{t,l}(\tau) + g^2 f_{b,k}(\tau)f_{b,l}(\tau) -$$

$$-f_{t,k}(0)f_{t,l}(0) - g^2 f_{b,k}(0)f_{b,l}(0))\frac{d\tau}{\tau} + O\left(\frac{\ln\frac{\Lambda^2}{m_t^2}}{\Lambda^2}\right). \tag{2.14}$$

The related integrals for $I_{kl}^{(1),(2)}$ have been calculated at large values of Λ, in the large-log limit $\ln(\Lambda^2/m_t^2) \simeq \ln(\Lambda^2/m_b^2) \gg 1$ and for the CP conserving phase. Further on we consider $g \ll 1$ as $m_b \ll m_t$. As well in the matrix elements containing large $\ln(\Lambda^2/m_t^2)$ we neglect isospin breaking effects. After substitution of (1.4) kinetic matrices $\hat{A}$ take the form:

$$A_{kl}^\sigma \simeq A_{kl}^\pi \simeq A_{kl}^{\pi^+} \simeq \begin{pmatrix} \left(4\ln\frac{\Lambda^2}{m_t^2} - \frac{23}{2}\right) & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{3}{2} \end{pmatrix} \tag{2.15}$$

Let us now obtain the matrix of second variations $\hat{B}^i$ of the effective potential for the Model I :

$$B_{kl}^\sigma = \frac{16\pi^2}{N_c}\frac{\partial^2}{\partial\sigma_k\partial\sigma_l}V_{eff}, \qquad B_{kl}^\pi = \frac{16\pi^2}{N_c}\frac{\partial^2}{\partial\pi_k\partial\pi_l}V_{eff},$$
$$B_{kl}^{\pi^+} = \frac{8\pi^2}{N_c}\frac{\partial^2}{\partial\pi_k^+\partial\pi_l^-}V_{eff}, \quad \text{for} \quad \Phi_k = \langle\Phi_k\rangle. \tag{2.16}$$

In the above approximation: $B_{kl}^{\pi^+} \simeq B_{kl}^\pi$ and therefore the spectra of charged and neutral pseudoscalars coincide. Other elements of matrices $\hat{B}$ are:

$$B_{11}^\sigma = -2\Delta_{11} + 48\phi_1^2\ln\left(\frac{\Lambda^2}{m_t^2}\right) - \frac{605}{4}\phi_1^2 - \frac{15\sqrt{3}}{2}\phi_1\phi_2 + \frac{9}{4}\phi_2^2,$$

$$B_{12}^\sigma = -2\Delta_{12} - \frac{15\sqrt{3}}{4}\phi_1^2 + \frac{9}{2}\phi_1\phi_2 + \frac{3\sqrt{3}}{4}\phi_2^2,$$

$$B_{22}^\sigma = -2\Delta_{22} + \frac{9}{4}\phi_1^2 + \frac{3\sqrt{3}}{2}\phi_1\phi_2 + \frac{27}{4}\phi_2^2,$$

$$B_{11}^\pi = -2\Delta_{11} + 16\phi_1^2\ln\left(\frac{\Lambda^2}{m_t^2}\right) - \frac{159}{4}\phi_1^2 - \frac{5\sqrt{3}}{2}\phi_1\phi_2 + \frac{3}{4}\phi_2^2,$$

$$B_{12}^\pi = -2\Delta_{12} - \frac{5\sqrt{3}}{4}\phi_1^2 + \frac{3}{2}\phi_1\phi_2 + \frac{\sqrt{3}}{4}\phi_2^2,$$

$$B_{22}^\pi = -2\Delta_{22} + \frac{3}{4}\phi_1^2 + \frac{\sqrt{3}}{2}\phi_1\phi_2 + \frac{9}{4}\phi_2^2. \tag{2.17}$$

For large $\ln(\Lambda^2/m_t^2) \gg 1$ both the solutions of (2.9) and the mass spectrum look differently in two regimes: $\Delta_{11} \sim m_t^2\ln(\Lambda^2/m_t^2) \gg \Delta_{12} \sim \Delta_{22} \sim m_t^2$ or $\Delta_{11} \sim \Delta_{12} \sim \Delta_{22} \sim m_t^2\ln(\Lambda^2/m_t^2) \gg m_t^2$.

In the first case the mass spectrum of physical scalar bosons H^0, h^0 reads:

$$m_{H^0}^2 = 8\phi_1^2 = 4m_t^2 \simeq \frac{\Delta_{11}}{\ln(\Lambda^2/m_t^2)};$$

$$m_{h^0}^2 = \frac{1}{2}\left(8\phi_2^2 + (\sqrt{3}\phi_1 + \phi_2)^2\right) - \frac{4}{3}\Delta_{22} \equiv \frac{3}{2}\delta m^2 - \frac{4}{3}\Delta_{22}, \tag{2.18}$$

6

and for physical pseudoscalar bosons $\pi, A, H^\pm$:

$$m_\pi^2 = 0; \qquad m_A^2 = m_{H^\pm}^2 = \frac{1}{6}\left(8\phi_2^2 + (\sqrt{3}\phi_1 + \phi_2)^2\right) - \frac{4}{3}\Delta_{22} \equiv \frac{1}{2}\delta m^2 - \frac{4}{3}\Delta_{22}, \quad (2.19)$$

where we have introduced the notation for the mass splitting between excited states:

$$m_{h^0}^2 - m_A^2 = \delta m^2. \tag{2.20}$$

One can see that, in general, $m_{h^0}^2 - m_A^2 = \delta m^2 \neq m_{H^0}^2 - m_\pi^2$ and the splitting between scalar and pseudoscalar states is not equidistant. But if the dynamical mass is a true constant, i.e. $\sqrt{3}\phi_1 + \phi_2 = 0$, then one recovers the remarkable relation:

$$m_{h^0}^2 - m_A^2 = m_{H^0}^2 - m_\pi^2 = 4m_t^2. \tag{2.21}$$

As $m_t^2 = 2\phi_1^2$ this parameter is fixed from the phenomenology and never takes zero values. Then for a given Δ_{22} the lowest masses of excited bosons arise at $\phi_1 = -3\sqrt{3}\phi_2$ and amount to:

$$\delta m_{min}^2 = \frac{1}{9}m_{H^0}^2; \qquad m_{h^0}^2 = \frac{1}{6}m_{H^0}^2 - \frac{4}{3}\Delta_{22} \geq \frac{1}{9}m_{H^0}^2;$$

$$m_A^2 = \frac{1}{18}m_{H^0}^2 - \frac{4}{3}\Delta_{22} \geq 0. \tag{2.22}$$

It follows from these inequalities that the second pseudoscalar Higgs boson can in principle be very light even massless whereas the second scalar Higgs particle can be lighter than the first one (of Nambu-Jona-Lasinio type) but still enough heavy.

For instance, if we treat these relations at the tree level then: $m_{H^0} = 2m_t \simeq 350 GeV$ but the lightest mass for the second Higgs boson $m_{h^0}^2 = \frac{1}{9}m_{H^0}^2 + m_A^2 \geq 130 GeV$ for $m_A \geq 50 GeV$ (see [33]).

In the case when $m_t^2 \ln\left(\Lambda^2/m_t^2\right) \sim \Delta_{11} \sim \Delta_{12} \sim \Delta_{22} \gg m_t^2$ the mass spectra can be easily derived from Eqs.(2.18), (2.19) retaining the last term, $m_{h^0}^2 \simeq m_A^2 \simeq -\frac{4}{3}\Delta_{22}$ and their splitting is again described by (2.20). Thus for such parameters Δ_{ik} the second multiplet of Higgs boson is indeed much heavier that the first one $m_{H^0}^2 \ll m_{h^0}^2 \simeq m_A^2 \sim m_t^2 \ln\left(\Lambda^2/m_t^2\right) \sim 1(TeV)^2$. Thus this scenario is practically equivalent to a One-Higgs SM for accessible energies. Therefore we will not consider it further on.

3. Chiral symmetry restoration in QCD-like models

In large N_c QCD-like models [34, 35] the leading contributions into two-point correlators of scalar and pseudoscalar quark densities are given by sums over an (infinite) number of meson poles:

$$\Pi_\sigma(p^2) = -\int d^4x \, \exp(ipx) \, \langle T\left(\bar{q}q(x)\,\bar{q}q(0)\right)\rangle$$

$$= \sum_n \frac{Z_n^\sigma}{p^2 + m_{\sigma,n}^2} + C_0^\sigma + C_1^\sigma p^2,$$

$$\Pi_\pi^{ab}(p^2) = \int d^4x \, \exp(ipx) \, \langle T \left(\bar{q}\gamma_5\tau^a q(x) \, \bar{q}\gamma_5\tau^b q(0) \right) \rangle$$

$$= \delta^{ab} \left(\sum_n \frac{Z_n^\pi}{p^2 + m_{\pi,n}^2} + C_0^\pi + C_1^\pi p^2 \right). \tag{3.1}$$

C_0 and C_1 are contact terms required for the regularization of infinite sums.

The high-energy asymptotics are given by perturbation theory and operator product expansion taking into account the asymptotic freedom of QCD-like interaction. As well the nonperturbative generation of (techni) gluon and quark condensates [24] is assumed to determine subleading power-like corrections to perturbative asymptotics.

The covariant derivative in a QCD-like gauge theory, $i\partial_\mu \to iD_\mu = i\partial_\mu + G_\mu$, contains gluon fields $G_\mu \equiv g_s\lambda^a G_\mu^a$, where $tr(\lambda^a\lambda^b) = 2\delta^{ab}$. The related gluon field strength is defined as $G_{\mu\nu} \equiv -i[D_\mu, D_\nu]$. g_s is the gauge coupling constant. Then the conventional QCD coupling constant is $\alpha_s \equiv g_s/4\pi$. Respectively, in the chiral limit ($m_q = 0$) the scalar and pseudoscalar correlators have the following behavior [24, 36, 37] at large p^2 (for one-flavor quarks) motivated by Operator Product Expansion:

$$\Pi_{\sigma,\pi}(p^2)|_{p^2\to\infty} \simeq \frac{N_c}{8\pi^2} \left(1 + \frac{11N_c\alpha_s}{8\pi} \right) p^2 \ln \frac{p^2}{\mu^2}$$
$$+ \frac{\alpha_s}{8\pi p^2} \langle (G_{\mu\nu}^a)^2 \rangle + \frac{\pi\alpha_s}{3p^4} \langle \bar{q}\gamma_\mu\lambda^k q\bar{q}\gamma_\mu\lambda^k q \rangle$$
$$\mp \frac{\pi\alpha_s}{2p^4} \langle \bar{q}\sigma_{\mu\nu}\lambda^k q\bar{q}\sigma_{\mu\nu}\lambda^k q \rangle + \mathcal{O}(\frac{1}{p^6}), \tag{3.2}$$

in Euclidean notations. Herein it is assumed that $\alpha_s(\mu) \ll 1$; $\mu > \Lambda_{CSB}$. In the large-N_c limit:

$$\left(\Pi_P(p^2) - \Pi_S(p^2) \right)_{p^2\to\infty} \equiv \frac{\Delta_{SP}}{p^4} + \mathcal{O}\left(\frac{1}{p^6}\right); \quad \Delta_{SP} \simeq 12\pi\alpha_s \left(<\bar{q}q> \right)^2, \tag{3.3}$$

where the vacuum dominance hypothesis [24] has been exploited to estimate four-quark condensate contribution in terms of bilinear quark condensates $<\bar{q}q>$. This rapidly decreasing asymptotics is a consequence of the chiral invariance of the lagrangean of a massless QCD-like theory.

When comparing (3.1) and the first term of (3.2) one can see that in order to reproduce the perturbative asymptotics the infinite number of resonances with the same quantum numbers should exist in each channel.

On the other hand, in the difference of scalar and pseudoscalar correlators the saturation may be successfully delivered by a finite number of low-lying resonances due to CSR [15, 23].

Thus CSR at high energies may be thought of as a possible constraint on the EQQM to be a manifestation of compositeness in the Higgs model [26, 38]. We shall

demand that, at the compositeness scale Λ_{CSB}, the relation (3.3) is approximately fulfilled.

Following the planar limit of the QCD-like interaction, eq.(3.1) one can make the two-resonance ansatz for scalar and pseudoscalar correlators provided by Two-channel EQQM model:

$$\Pi_\sigma(p) = \frac{Z_\sigma}{p^2 + m_\sigma^2} + \frac{Z_{\sigma'}}{p^2 + m_{\sigma'}^2} + C_0^\sigma;$$

$$\Pi_\pi(p) = \frac{Z_\pi}{p^2} + \frac{Z_{\pi'}}{p^2 + m_{\pi'}^2} + C_0^\pi. \tag{3.4}$$

We remark that for this type of models the constants can be taken $C_1^\sigma = C_1^\pi = 0$. From the requirement of asymptotic CSR (3.3) it follows that:

$$C_0^\sigma = C_0^\pi \equiv C \left(= \frac{<\bar{q}q>}{m_{dyn}} < 0 \right); \tag{3.5}$$

$$Z_\sigma + Z_{\sigma'} = Z_\pi + Z_{\pi'}; \qquad Z_\sigma m_\sigma^2 + Z_{\sigma'} m_{\sigma'}^2 = Z_{\pi'} m_{\pi'}^2 + \Delta_{SP}. \tag{3.6}$$

The first two relations can be fulfilled in the conventional NJL model which corresponds to the one-resonance ansatz, $Z_{\sigma',\pi'} = 0$, whereas the last one can be provided only in a two-resonance model, for the Δ_{SP} in (3.3) (see below).

In order to apply the above CSR sum rules we have to obtain the appropriate two-point correlators for scalar and pseudoscalar composite Higgs fields.

4. CSR sum rules in 2HQQM

Let us introduce in 2HQQM the external sources which couple to the scalar and pseudoscalar quark densities. With their help one can easily derive required quark correlators. As these densities are doublets in our model , eq.(1.2), the relevant sources $X_1 = (\chi_{1j}), X_2 = (\chi_{2j})$ are taken as doublets as well. The sources are complex: $\chi_{kj} = S_{kj} + iP_{kj}$, generating both scalar and pseudoscalar densities. The structure of the corresponding operators in the quark lagrangean is designed similar to the Yukawa operators in (2.3):

$$\mathcal{L}_I(S, P) = i \sum_{k=1}^{2} \left[g_{t,k} \widetilde{X}_k^\dagger J_{t,k} + g_{b,k} X_k^\dagger J_{b,k} \right] + h.c. \tag{4.1}$$

We remind that in 2HQQM under consideration: $g_{t,k} = 1 \gg g_{b,k} = g$ and take for simplicity $g = 0$. Then the effect of external sources can be separated by shifting the Higgs fields in the quark part of the lagrangean (2.3):

$$\phi_{kj} \longrightarrow \bar{\phi}_{kj} + \chi_{kj}. \tag{4.2}$$

9

The dynamical boson fields arise as fluctuations around the solutions of the mass-gap equation (2.9):

$$\bar{\phi}_{k1} = \phi_{k1} - \chi_{k1}; \quad \bar{\phi}_{k2} = \phi_{k2} - \chi_{k2} + <\bar{\phi}_{k2}> . \tag{4.3}$$

In terms of doublets X_k, Φ_k the supplementary lagrangean takes the form:

$$\Delta\mathcal{L}_I = N_c\Lambda^2 \sum_{k,l=1}^{2} \left[X_k^\dagger(a^{-1})_{kl}X_l - X_k^\dagger(a^{-1})_{kl}\Phi_l \right.$$
$$\left. -\Phi_k^\dagger(a^{-1})_{kl}X_l - X_k^\dagger(a^{-1})_{kl}\langle\Phi_l\rangle - \langle\Phi_k^\dagger\rangle(a^{-1})_{kl}X_l \right]. \tag{4.4}$$

The terms linear in external sources are irrelevant for correlators and will be neglected further on. As we study the CP conserving phase with non-trivial v.e.v. for scalar fields only the physical parameters of neutral and charged pseudoscalar bosons are the same. Therefore we can restrict ourselves with neutral components only. Then with notations:

$$\phi_{k2} = \frac{1}{\sqrt{2}}(\sigma_k + i\pi_k); \qquad \chi_{k2} = \frac{1}{\sqrt{2}}(S_k + i\,P_k), \tag{4.5}$$

one obtains the quadratic part of the lagrangean $\mathcal{L}_I$ consisted of (2.11) and (4.4). As it is gaussian one can easily unravel the dependence on external fields with the help of classical Eqs. of motion:

$$\sigma_k^{\text{cl}} = 16\pi^2\Lambda^2 \sum_{l,m=1}^{2} \left(A^{\sigma\sigma}p^2 + B^{\sigma\sigma}\right)_{kl}^{-1} a_{lm}^{-1}S_m \simeq 2\Lambda^2 \sum_{l=1}^{2} \left(A^{\sigma\sigma}p^2 + B^{\sigma\sigma}\right)_{kl}^{-1} S_l$$

$$\pi_k^{\text{cl}} = 16\pi^2\Lambda^2 \sum_{l,m=1}^{2} \left(A^{\pi\pi}p^2 + B^{\pi\pi}\right)_{kl}^{-1} a_{lm}^{-1}P_m \simeq 2\Lambda^2 \sum_{l=1}^{2} \left(A^{\pi\pi}p^2 + B^{\pi\pi}\right)_{kl}^{-1} P_l, \tag{4.6}$$

which is simplified in the vicinity of polycritical point, $\quad 8\pi^2 a_{kl}^{-1} \simeq \delta_{kl}$.

The resulting effective action for generating of two-point correlators is given by:

$$S^{(2)} \simeq \frac{N_c\Lambda^2}{16\pi^2} \sum_{k,l=1}^{2} \left(S_k\Gamma_{kl}^{(\sigma)}S_l + P_k\Gamma_{kl}^{(\pi)}P_l \right)$$

$$\Gamma_{kl}^{(\sigma)} = \delta_{kl} - 2\Lambda^2 \left(A^{\sigma\sigma}p^2 + B^{\sigma\sigma}\right)_{kl}^{-1}; \quad \Gamma_{kl}^{(\pi)} = \delta_{kl} - 2\Lambda^2 \left(A^{\pi\pi}p^2 + B^{\pi\pi}\right)_{kl}^{-1}. \tag{4.7}$$

When taking the approximate expressions (2.15), (2.17) for matrices $\hat{A}$, $\hat{B}$ one derives the inverse propagators. In particular, the strictly local quark density can be presented as a superposition of two currents:

$$\bar{t}t = \frac{1}{2}(\bar{t}f_1t - \sqrt{3}\bar{t}f_2t), \tag{4.8}$$

and in the scalar channel the related correlator (3.1) has the following form:

$$\Pi_\sigma(p^2) = -\frac{N_c\Lambda^2}{16\pi^2}\left[\Gamma_{11}^{(\sigma)} + 3\Gamma_{22}^{(\sigma)} - 2\sqrt{3}\Gamma_{12}^{(\sigma)}\right] = C^\sigma + \frac{Z_{H^0}}{p^2 + m_{H^0}^2} + \frac{Z_{h^0}}{p^2 + m_{h^0}^2};$$

$$C^\sigma = -\frac{N_c\Lambda^2}{4\pi^2} \simeq \frac{<\bar{t}t>}{m_t};$$

$$Z_{H^0} = \frac{N_c\Lambda^4}{48\pi^2\left(\ln\frac{\Lambda^2}{m_t^2} - 3\right)\left(m_{H^0}^2 - m_{h^0}^2\right)}$$

$$\times\left[12\left(\ln\frac{\Lambda^2}{m_t^2} - 3\right)m_{H^0}^2 + 6\Delta_{11} + 4\sqrt{3}\Delta_{12} + 2\Delta_{22} - 144\phi_1^2\ln\frac{\Lambda^2}{m_t^2} + 474\phi_1^2\right.$$

$$\left. + 12\sqrt{3}\phi_1\phi_2 - 18\phi_2^2\right] = O\left(\frac{1}{\ln\frac{\Lambda^2}{m_t^2}}\right);$$

$$Z_{h^0} = -Z_{H^0} + \frac{N_c\Lambda^4}{4\pi^2}; \tag{4.9}$$

and similarly in the pseudoscalar channel,

$$\Pi_\pi(p^2) = -\frac{N_c\Lambda^2}{16\pi^2}\left[\Gamma_{11}^{(\pi)} + 3\Gamma_{22}^{(\pi)} - 2\sqrt{3}\Gamma_{12}^{(\pi)}\right] = C^\pi + \frac{Z^\pi}{p^2} + \frac{Z_A}{p^2 + m_A^2};$$

$$C^\pi = C^\sigma = -\frac{N_c\Lambda^2}{4\pi^2} \simeq \frac{<\bar{t}t>}{m_t};$$

$$Z^\pi = -\frac{N_c\Lambda^4}{48\pi^2\left(\ln\frac{\Lambda^2}{m_t^2} - 3\right)m_A^2}\left[2\Delta_{22} + \Delta_{12}\left(4\sqrt{3} - 6\frac{\phi_2}{\phi_1}\right) + \frac{27}{4}\phi_1^2 - \frac{29\sqrt{3}}{4}\phi_1\phi_2\right.$$

$$\left. + \frac{3}{4}\phi_2^2 + \frac{3\sqrt{3}}{4}\frac{\phi_2^3}{\phi_1}\right] = O\left(\frac{1}{\ln\frac{\Lambda^2}{m_t^2}}\right);$$

$$Z_A = -Z_\pi + \frac{N_c\Lambda^4}{4\pi^2}. \tag{4.10}$$

We stress that the residues in poles are of different order of magnitude:

$$Z_{H^0} \sim Z^\pi \sim \frac{1}{\ln\frac{\Lambda^2}{m_t^2}} \ll Z_{h^0} \sim Z_A, \tag{4.11}$$

which can be seen from the first Eq. in (4.9) after using the mass-gap Eqs.(2.9) and taking the value (2.18) of the scalar boson mass, $m_{H^0}^2 \simeq 8\phi_1^2$. Indeed then the logarithms cancel each other in the combination, $12m_{H^0}^2\ln\frac{\Lambda^2}{m_t^2}m_{H^0}^2 + 6\Delta_{11} - 144\phi_1^2\ln\frac{\Lambda^2}{m_t^2}$.

Now we are able to impose and check the CSR constraints (3.6). First of all one can see that the chiral symmetry is not broken in leading asymptotics, $C^\pi = C^\sigma$. Next we check the subleading asymptotics which represents the generalized σ-model relation:

$$Z_{H^0} + Z_{h^0} = Z_\pi + Z_A = \frac{N_c\Lambda^4}{4\pi^2}, \tag{4.12}$$

11

which is fulfilled, in fact, at a high precision including subleading $1/\ln \Lambda^2$ terms.

The last constraint in (3.6) can be satisfied for an appropriate 4-fermion condensate contribution Δ_{SP}. Thus:

$$Z_{H^0} m_{H^0}^2 + Z_{h^0} m_{h^0}^2 = Z_A m_A^2 + \Delta_{SP}. \tag{4.13}$$

Taking into account (4.11) and the sum rule (4.12) one concludes that:

$$Z_{h^0} \simeq Z_A \simeq \frac{N_c \Lambda^4}{4\pi^2}; \qquad \frac{N_c \Lambda^4}{4\pi^2}\left(m_{h^0}^2 - m_A^2\right) = \Delta_{SP}. \tag{4.14}$$

In order to estimate a maximal scale of compositeness $\Lambda \equiv \Lambda_C$ due to QCD-like forces we assume that presumably the typical (techni) QCD scale is of order $\Lambda_{TQCD} \sim m_t \ll \Lambda_C$ and one can apply effectively the one-loop approximation to calculate the strong interaction constant α_s (see, [33]). As it follows from (2.22), the minimal value of mass splitting corresponds to $m_{H^0}^2/9$. Respectively the value of Δ_{SP} is described by the relation (3.3) and in the present 2HQQM the condensates are given in (4.9). Let us combine these estimates and relations to derive the upper bound on the compositeness scale Λ_C:

$$\frac{N_c \Lambda_C^4 m_t^2}{9\pi^2} = \min(\Delta_{SP}) = 12\pi\alpha_s \frac{N_c^2 \Lambda_C^4 m_t^2}{16\pi^4}; \tag{4.15}$$

hence:

$$\alpha_s N_c \simeq \frac{12\pi}{11\left(1 - \frac{2N_f}{N_c}\right)\ln\frac{\Lambda_C^2}{m_t^2}} > \frac{4\pi}{27} \quad \text{or} \quad \Lambda_C \leq 10^4 GeV. \tag{4.16}$$

Thereby with the help of CSR sum rules we have showed that there is a certain window to have relatively light composite Higgs bosons in the effective 2HQQM created by a more fundamental QCD-like theory.

For a comparison we also check the consistency of the last sum rule (4.13) in the one-channel top-condensate SM. One deals then with only one scalar,H^0, and a triplet of Goldstone bosons, π^a. Respectively, in the sum rules (3.6), (4.13) one should retain only Z^π, Z_{H^0}. Hence they read:

$$Z^\pi = Z_{H^0} = \frac{N_c \Lambda^4}{16\pi^2 \ln\frac{\Lambda^2}{m_t^2}};$$

$$Z_{H^0} m_{H^0}^2 = \frac{1}{4}\frac{N_c \Lambda_C^4 m_t^2}{\pi^2 \ln\frac{\Lambda_C^2}{m_t^2}} \neq \Delta_{SP} = \frac{9}{11}\frac{N_c \Lambda_C^4 m_t^2}{\pi^2 \ln\frac{\Lambda_C^2}{m_t^2}}, \tag{4.17}$$

where eqs.(4.7) with $A \simeq A_{11}, B \simeq B_{11}$ from (2.15), (2.17) are employed, the Nambu relation $m_{H^0}^2 = 4m_t^2$ is used and the correlators are derived according to eqs.(4.9), (4.10). As well the definition (3.3) for Δ_{SP} and the expression (4.16) for α_s is inserted. Evidently there is a serious discrepancy between the left and right sides of the last sum rule which is remarkably independent on scales, colors and flavours. Thus it happens to be impossible to embed the one-channel Top-mode SM into a QCD-like underlying theory.

5. Conclusions

1.   We have found that in the framework of 2HQQM of type I two composite Higgs doublets can be created dynamically as a consequence of DCSB in two channels. Lighter Higgs bosons appear as radial excited states in the language of the Potential Quark Model.

2.   We have proved that CSR at high energies is realized in the 2HQQM of type I in the Nambu-Jona-Lasinio phase near tricritical point. Thereby these models in the NJL phase can be regarded as effective models originating from a QCD-like underlying theory with an electroweak compositeness scale of order $1 \div 10 TeV$ (it may be also a fundamental, string scale [39] when, at least, additional five dimensions of inverse size 10 Mev exist).

3.   One can show that in other phases explored in two-channel models in [17, 26], such as the anomalous or special phase, the last CSR constraint cannot be fulfilled for *any* choice of parameters. It means that such phases cannot be realized in effective models motivated by underlying QCD-like fundamental theory. But the question remains open about what kind of underlying theory could induce such phases at lower energies.

4.   It turns out that the soft-momentum limit of the correlators (3.4) is connected to the structural constants of the effective chiral lagrangian [40, 41] (in our case of the EW lagrangian in the large Higgs mass limit [42]):

$$\frac{Z_\sigma}{m_\sigma^2} + \frac{Z_{\sigma'}}{m_{\sigma'}^2} + C = 8B_0^2(2L_8 + H_2);$$

$$\frac{Z_{\pi'}}{m_{\pi'}^2} + C = 8B_0^2(-2L_8 + H_2). \tag{5.1}$$

One can think about how to apply these relations together with the empirical estimations on the corresponding structural coupling constant for the determination of the size of the (techni-,top-)quark condensate $< \bar{q}q >= -B_0 F_0^2$ (with F_0 being an electroweak scale).

5.   Since the first paper [10] on the compositeness due to strong 4-fermion forces it is known [11, 12, 13] that the low-mass spectrum is provided by a fine-tuning. It is remarkable that the above pattern for heavy Higgs particles can be put in certain correspondence to the predictions found with the help of the modified Veltman fine-tuning hypothesis [43].

6. In this paper we have not considered the CP and electroneutrality violating scenario for 2HQQM in details although it may occur as an intermediate stage in the

formation of baryon assymetry of the Universe [31, 44].

7.   There exists another way to build multi-channel (nonlocal but separable) quark models [45, 46] and it is of certain interest to develop the similar CSR analysis in that approach.

8.   More realistic predictions for masses and coupling constants should be based, of course, on the full SM action including vector bosons and on the RG improved calculations of low-energy parameters both in Higgs and in fermion sectors which are in preparation.

Acknowledgments

References

[1] A. A. Anselm, N. G. Uraltsev, V. A. Khoze, *Sov. Phys. Usp.* **28** (1985) 113;
     A.A. Anselm, *Surveys High Energ.Phys.* **7** (1994) 107.

[2] T.D.Lee, *Phys. Rev.* **D 8** (1973) 1226.

[3] J.F.Gunion, H.E.Haber, G.Kane, S.Dawson, *The Higgs Hunter's Guide*, Addison-Wesley 1990.

[4] S. L. Glashow, S. Weinberg, *Phys. Rev.* **D 15** (1977) 1958.

[5] D. Atwood, L. Reina, A. Soni, *Phys. Rev.* **D 55** (1997) 3156.

[6] G. Kreyerhoff, R. Rodenberg, *Phys. Lett.* **B 226** (1989) 323;
     J. Freund, G. Kreyerhoff, R. Rodenberg, *Phys. Lett.* **B 280** (1992) 267.

[7] M. Sher, *Phys. Rep.* **179** (1989) 273;
     S. Nie, M. Sher, `hep-ph/9811234`.

[8] J. Erler, P. Langacker, `hep-ph/9809352`.

[9] G. Degrassi et al, *Phys. Lett.* **B 418** (1998) 209;
     G. D'Agostini, G. Degrassi, `hep-ph/9902226`.

[10] Y.Nambu, G.Jona-Lasinio, *Phys. Rev.* **122** (1961) 345.

[11] V. A. Miransky, M. Tanabashi, K. Yamawaki, *Mod. Phys. Lett.* **A 4** (1989) 1043; *Phys. Lett.* **B 221** (1989) 177.

[12] W. J. Marciano, *Phys. Rev. Lett.* **62** (1989) 2793; *Phys. Rev.* **D 41** (1990) 219.

[13] W.A.Bardeen, C.T.Hill, M.Lindner,*Phys. Rev.* **D 41** (1990) 1647.

[14] M. Lindner, D. Lüst, *Phys. Lett.* **B 272** (1991) 91; M. Lindner, *Int. J. Mod. Phys.* **A 8** (1993) 2167.

[15] A. A. Andrianov, V. A. Andrianov, `hep-ph/9705364`; *Zapiski Nauch. Sem. POMI (Proc. Steklov Math. Inst., St. Petersburg, Russia)*, **245**, No.14 (1996) 5.

[16] M. Krawczyk, `hep-ph/9803484`; `hep-ph/9812536` and refs. therein.

[17] A.A.Andrianov and V.A.Andrianov, *Int. J. Mod. Phys.* **A 8** (1993) 1981; *Theor. Math. Phys.* **93** (1992) 1126; *Theor. Math. Phys.***94** (1993) 3; `hep-ph/9309297`; *Proc. School-Sem. "Hadrons and nuclei from QCD", Tsuruga/Vladivostok/Sapporo 1993*, WSPC (1994) 341; *Zap. Nauch. Sem. POMI (Proc. Steklov Math.Inst.,St. Petersburg, Russia)*, **209**, No.12 (1994) 3; *Nucl. Phys.* **39B,C** *(Proc. Suppl.)* (1995) 257.

[18] K. G. Wilson, J. B. Kogut,*Phys. Rep.* **C12** (1974) 75.

[19] A. A. Andrianov, V. A. Andrianov, V. L. Yudichev, R. Rodenberg, `hep-ph/9709475` ; *Int. J. Mod. Phys.* **A 14** (1999) 323.

[20] T. L. Barklow *et al.*, *1996DPF/DPB Summer Study, Snowmass, Colorado*, `hep-ph/9704217`.

[21] T. Appelquist, J. Terning, L. C. R. Wijewardhana. *Phys. Rev. Lett.* **79** (1997) 2767 and references therein.

[22] R. S. Chivukula, B. A. Dobrescu, H. Georgi, C. T. Hill. `hep-ph/9809470` and references therein.

[23] A. A. Andrianov, D. Espriu, R. Tarrach, *Nucl. Phys.* **B 533** (1998) 429.

[24] M. A. Shifman, A. I. Vainstein, V. I. Zakharov, *Nucl. Phys.* **B 147** (1979) 385, 448.

[25] G. Cvetic, `hep-ph/9702381` and references therein.

[26] A.A.Andrianov, V.A.Andrianov, V.L.Yudichev, `hep-ph/9512256`, *Proc. X Int. Worksh. on HEP and QFT (Zvenigorod, Russia, Sept. 1995)*, MSU Publ. (1996) 211; `hep-ph/9610376`, *Proc. XI Int. Worksh. on HEP and QFT (St.Petersburg, Sept. 1996)*, MSU Publ. (1997) 160; *Theor. Math. Phys.* **108** (1996) 1069.

[27] G. Cvetic, C. S. Kim, S. S. Hwang, *Phys. Rev.* **D 58** (1998) 116003.

[28] G. C. Branco, *Phys. Rev.* **D 22** (1980) 2901;
G.C. Branco, J.-M. Gerard, W. Grimus, *Phys. Lett.* **B 136** (1984) 383.

[29] A. Skjold, P. Osland, *Nucl. Phys.* **B 453** (1995) 3;
C.A. Boe et al. `hep-ph/9811505`.

[30] W. Bernreuter, `hep-ph/9808453` and refs. therein;
W. Bernreuter, A. Brandenburg, M. Flesch,*Phys. Rev.* **D 56** (1997) 90;
`hep-ph/9812387`.

[31] A. B. Lahanas, V.C. Spanos, V. Zarikas, `hep-ph/9812535`.

[32] R. Rodenberg, *Int. J. Mod. Phys.* **A 11** (1996) 4779 and references therein.

[33] C.Caso *et al.* (PDG), *European Phys. J.* **C3** (1998) 1.

[34] G. t'Hooft, *Nucl. Phys.* **B 72** (1974) 461.

[35] E.Witten, *Nucl. Phys.* **B 160** (1979) 57.

[36] L. J. Reinders, H. Rubinstein, S. Yazaki, *Phys. Rep.* **127** (1985) 1 and references therein.

[37] M. Jamin, M. Münz, *Z. Physik* **C 66** (1995) 633.

[38] N. V. Krasnikov, R. Rodenberg, *Nuovo Cim.* **A105** (1992) 1601.

[39] T. Banks, A. Nelson, M. Dine, `hep-th/9903019`.

[40] J. Gasser, H. Leutwyler. *Nucl. Phys.* **B 250** (1985) 465.

[41] J. Bijnens, E. de Rafael, H. Zheng, *Z. Physik* **C 62** (1994) 437;
J. Bijnens, *Phys. Rep.* **265** (1996) 369.

[42] A. Dobado, D. Espriu, M. J. Herrero, *Phys. Lett.* **B 255** (1991) 405;
P. Ciafaloni , D. Espriu , *Phys. Rev.* **D 56** (1997) 1752.

[43] A.A.Andrianov, R. Rodenberg and N. V. Romanenko, *Nuovo Cim.* **108A** (1995) 577.

[44] M. Laine, K. Rummukainen, `hep-ph/9811369`.

[45] M. K. Volkov, C. Weiss, *Phys. Rev.* **D 56** (1997) 221;
M. K. Volkov, D. Ebert, M. Nagy, `hep-ph/9705334`.

[46] A. A. Andrianov, V. A. Andrianov, M. K. Volkov, V. L. Yudichev,
JINR Rapid Commun. **4 - 90** (1998) 45.